Name ANSWERS	Class

MATHS TEACHER HUB

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Functions

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

9 The functions f and g are such that

$$f(x) = 3(x - 4) \quad \text{and} \quad g(x) = \frac{x}{5} + 1$$

(a) Find the value of f(10)

$f(10) = 3(10-4)$

$= 18$

18

(1)

(b) Find $g^{-1}(x)$

$g^{-1}(x) =$ $5(x-1)$

(2)

(c) Show that $ff(x) = 9x - 48$

$f(x) = 3(x-4)$

$ff(x) = 3[3(x-4) - 4]$

$= 3[3x - 12 - 4]$

$= 3[3x - 16]$

$= 9x - 48$

(2)

(Total for Question 9 is 5 marks)

10 $f(x) = 4\sin x°$

(a) Find f(23)
Give your answer correct to 3 significant figures.

$f(23) = 4\sin(23)$

$= 1.5629$

1.56

(1)

$g(x) = 2x - 3$

(b) Find fg(34)
Give your answer correct to 3 significant figures.

$g(34) = 2(34) - 3$

$= 65$

$fg(34) = 4\sin(65)$

$= 3.625231148$

3.63

(2)

$h(x) = (x + 4)^2$

Ivan needs to solve the following equation $h(x) = 25$

He writes

$(x + 4)^2 = 25$
$x + 4 = 5$
$x = 1$

$\sqrt{25} = \pm 5$

This is not fully correct.

(c) Explain why.

Ivan should have the positive and negative roots of 25

(1)

(Total for Question 10 is 4 marks)

10 The function f is such that

$$f(x) = 4x - 1$$

(a) Find $f^{-1}(x)$

$f^{-1}(x) = \frac{x+1}{4}$

(2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

(b) work out the value of k

$fg(x) = 4(kx^2) - 1$

$= 4kx^2 - 1$

$= 4k(2)^2 - 1$

$= 16k - 1$

$16k - 1 = 12$

$16k = 13$

$k = \frac{13}{16}$

$k = \frac{13}{16}$

(2)

(Total for Question 10 is 4 marks)

11 f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find f(–5)

$f(-5) = \frac{2}{(-5)^2}$

$\frac{2}{25}$

(1)

(b) Find fg(1)

$g(1) = 4(1)^3$

$= 4$

$f(4) = \frac{2}{4^2} = \frac{2}{16}$

$\frac{1}{8}$

(2)

(Total for Question 11 is 3 marks)

16 The functions f and g are given by

$$f(x) = \frac{12}{x+1} \quad \text{and} \quad g(x) = 5 - 3x$$

(a) Find $f(-3)$

$$f(-3) = \frac{12}{-3+1}$$

$$= \frac{12}{-2}$$

-6

(1)

(b) Find $fg(1)$

$$g(1) = 5 - 3(1)$$

$$= 2$$

$$f(2) = \frac{12}{2+1}$$

$$= 4$$

4

(2)

(c) Find $g^{-1}(4)$

$$g^{-1}(x) = \frac{5-x}{3}$$

$$g^{-1}(4) = \frac{5-4}{3}$$

$$= \frac{1}{3}$$

$\frac{1}{3}$

(2)

(Total for Question 16 is 5 marks)

18 The function f is given by

$$f(x) = 2x^3 - 4$$

(a) Show that $f^{-1}(50) = 3$

$f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}}$

$f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}}$

$= \sqrt[3]{\frac{54}{2}} \quad = \sqrt[3]{27} = 3$

(2)

The functions g and h are given by

$$g(x) = x + 2 \text{ and } h(x) = x^2$$

(b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

$hg(x) = (x+2)^2$

$hg(x) = x^2+4x+4$

$x^2+4x+4 = 3x^2+x-1$

$0 = 2x^2-3x-5$

$0 = (2x-5)(x+1)$

$x = \frac{5}{2} \quad x = -1$

$x = 2.5 \quad x = -1$

(4)

(Total for Question 18 is 6 marks)

18 $f(x) = 3x^2 - 2x - 8$

Express $f(x + 2)$ in the form $ax^2 + bx$

$f(x+2) = 3(x+2)^2 - 2(x+2) - 8$

$= 3(x^2+4x+4) - 2x - 4 - 8$

$= 3x^2+12x+12 - 2x - 12$

$= 3x^2 + 10x$

$3x^2+10x$

(Total for Question 18 is 3 marks)

19 The functions f and g are such that

$$f(x) = (2x + 3)^2 \quad \text{and} \quad g(x) = 2x - 1$$

(a) Find $gf(-3)$

$f(-3) = [2(-3) + 3]^2$

$= [-6 + 3]^2$

$= [-3]^2$

$= 9$

$g(9) = 2(9) - 1$

$= 18 - 1$

17

(2)

(b) Find $g^{-1}(x)$

$g^{-1}(x) = \frac{x+1}{2}$

(2)

(Total for Question 19 is 4 marks)

19 The functions g and h are such that

$$g(x) = \sqrt[3]{2x - 5} \qquad h(x) = \frac{1}{x}$$

(a) Find g(16)

$g(16) = \sqrt[3]{2(16)-5}$

$= \sqrt[3]{32-5}$

$= \sqrt[3]{27}$

3

(1)

(b) Find $hg^{-1}(x)$

Give your answer in terms of x in its simplest form.

$g(x) = \sqrt[3]{2x-5}$

$g^{-1}(x) = \frac{x^3+5}{2}$

$= \frac{1}{2}(x^3+5)$

$hg^{-1}(x) = \frac{1}{\frac{1}{2}(x^3+5)}$

$= \frac{2}{x^3+5}$

$hg^{-1}(x) =$ $\frac{2}{x^3+5}$

(3)

(Total for Question 19 is 4 marks)

19 f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find g(5)

$g(5) = 3[2(5)+1]$

$= 3[10+1]$

$= 3[11)$

33

(1)

(b) Find gf(9)

$f(9) = \frac{12}{\sqrt{9}}$

$= \frac{12}{\pm 3} = \pm 4$

$g(\pm 4) = 3[2(\pm 4)+1]$

$= 3[\pm 8 + 1]$

$= 3[9]$ or $3[-7]$

27 or -21

(2)

(c) Find $g^{-1}(6)$

$g(x) = 6x+3$

$g^{-1}(x) = \frac{x-3}{6}$

$g^{-1}(6) = \frac{6-3}{6} = \frac{3}{6}$

$\frac{1}{2}$

(2)

(Total for Question 19 is 5 marks)

19 For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

$gf(x) = 2[(x+1)^2 - 1]$

$= 2[x^2 + 2x + 1 - 1]$

$= 2[x^2 + 2x]$

$= 2x^2 + 4x$

$= 2x(x+2)$

(2)

(b) Find $g^{-1}(7)$

$g(x) = 2x - 2$

$g^{-1}(x) = \frac{x+2}{2}$

$g^{-1}(7) = \frac{7+2}{2}$

$= \frac{9}{2}$

4.5

(2)

(Total for Question 19 is 4 marks)

20 For $x \geqslant 0$, the functions f and g are such that

$$f(x) = 3x + 4 \qquad g(x) = \frac{\sqrt{x} + 2}{5}$$

(a) Find $g^{-1}(x)$

$g^{-1}(x) =$ $(5x-2)^2$

(2)

(b) Solve $gf(x) = 3$

$$gf(x) = \frac{\sqrt{3x+4} + 2}{5} = 3$$

$$\sqrt{3x+4} + 2 = 15$$

$$\sqrt{3x+4} = 13$$

$$3x+4 = 169$$

$$3x = 165$$

$$x = 55$$

$x =$ 55

(3)

(Total for Question 20 is 5 marks)

21 The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \qquad \text{and} \qquad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out gf(1)

$f(1) = 3(1)^2 + 1$

$= 3 + 1$

$= 4$

$g(4) = \frac{4}{4^2}$

$= \frac{4}{16}$

$\frac{1}{4}$

(2)

The function h is such that $h = (fg)^{-1}$

(b) Find $h(x)$

$fg(x) = 3\left(\frac{4}{x^2}\right)^2 + 1$

$= 3\left(\frac{16}{x^4}\right) + 1$

$= \frac{48}{x^4} + 1$

$x = \frac{48}{y^4} + 1$

$x - 1 = \frac{48}{y^4}$

$y^4(x-1) = 48$

$y^4 = \frac{48}{(x-1)}$

$y = \sqrt[4]{\frac{48}{x-1}}$

$\sqrt[4]{\frac{48}{x-1}}$

(4)

(Total for Question 21 is 6 marks)

21 The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$f^{-1}(x) = \dfrac{x+1}{3}$

(2)

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$$fg(x) = 3(x^2+4) - 1$$

$$= 3x^2 + 12 - 1$$

$$= 3x^2 + 11$$

$$2gf(x) = 2\left[(3x-1)^2 + 4\right]$$

$$= 2\left[(9x^2 - 6x + 1) + 4\right]$$

$$= 18x^2 - 12x + 10$$

$$3x^2 + 11 = 18x^2 - 12x + 10 \qquad (-3x^2)$$

$$11 = 15x^2 - 12x + 10 \qquad (-3x^2)$$

$$0 = 15x^2 - 12x - 1 \qquad (-11)$$

(5)

(Total for Question 21 is 7 marks)

22 $f(x) = \sqrt[3]{x}$

$g(x) = 2x + 3$

$h(x) = fg(x)$

Find $h^{-1}(x)$

$fg(x) = \sqrt[3]{2x+3}$

$h(x) = \sqrt[3]{2x+3}$

$h^{-1}(x) = \dfrac{x^3-3}{2}$

$h^{-1}(x) =$ $\dfrac{x^3-3}{2}$

(Total for Question 22 is 3 marks)

22 The functions f and g are such that

$f(x) = 5x + 3$ $g(x) = ax + b$ where a and b are constants.

$g(3) = 20$ and $f^{-1}(33) = g(1)$

Find the value of a and the value of b.

$g(3) = 20$

$g(3) = 3a + b$

$\therefore 3a + b = 20$

$f(x) = 5x + 3$

$f^{-1}(x) = \frac{x-3}{5}$

$g(x) = ax + b$

$g(1) = a(1) + b$

$= a + b$

$f^{-1}(33) = \frac{33-3}{5} = 6$

$f^{-1}(33) = g(1)$

$6 = a + b$

$3a + b = 20$

$- \quad a + b = 6$

$2a = 14$

$a = 7$

$a = 7$

$b = -1$

(Total for Question 22 is 5 marks)